THE CIRCLE

Supplies: notebook, scissors, construction paper, compass, protractor, pencil.

Curriculum (1 hour, 30 minutes)

1. The circle often has a mystical meaning and represents the eternal, infinity, or the Sun.
2. Let's examine the mathematics of the circle:
   a. Definition of the circle
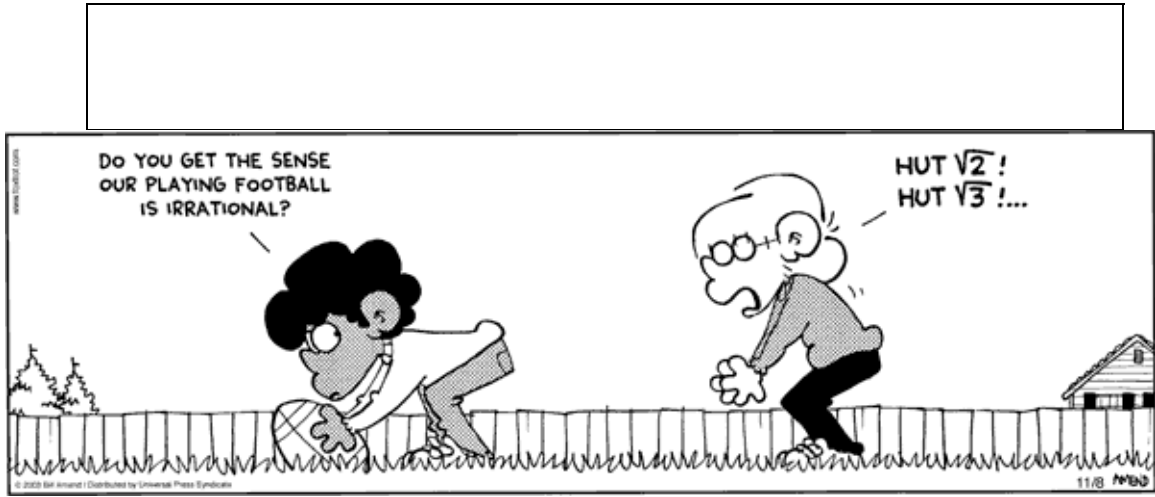
   ┌───┐
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   b. The radius of the circle

   ┌───┐
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   c. The diameter of the circle

   ┌───┐
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   d. The circumference of the circle – the amazing number $\pi \approx 3.1415926...$
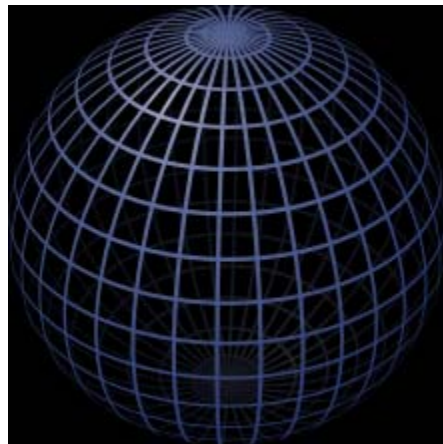
   (Activity: plot a circle with a cord and pencil, note the radius and diameter, wrap the cord around the circle, how many times the diameter fits the circumference?)

3. Rational and irrational numbers.

Examples: integer numbers and fractions are rational, square root of two is not.

4. Circles on the surface of the Earth

Question: if the circumference and diameter is measured on the surface of a sphere, what is the smallest ratio of the circumference to the diameter?

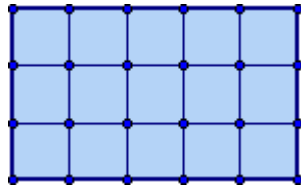Question: can a person walk 1 mile South, then 1 mile East, then 1 mile West, and end up at the starting point?

5. Putting a belt around the Earth and around an apple.

Activity: cord around an apple and around a group of 5 children

Calculation: $2\pi(r + 1) = 2\pi r + 2\pi \approx 2\pi r + 6$

6. Area of a circle.
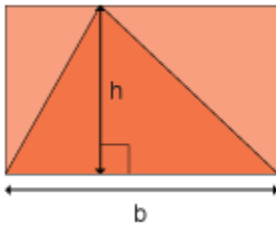 a. Area of a rectangle

  b. Area of a parallelogram
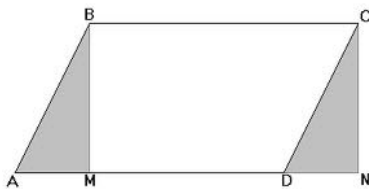
Activity: cut a triangle out of a rectangular piece of construction paper, add to the other side.
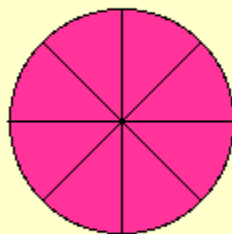
Question: what if the parallelogram is too skewed?

c. Area of a triangle

Activity: an alternative way, using two triangles and parallelogram.

d. Area of a circle

Area = πr^2

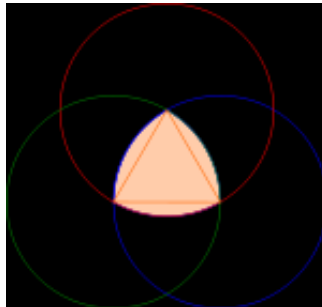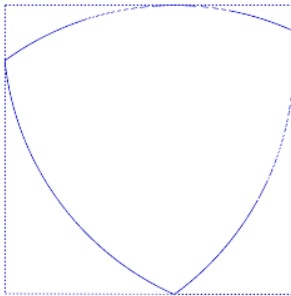Activity: Cut the circle drawn earlier like pizza and put the pieces together.

7. Why are manhole covers circular?

8. Other shapes with constant width:
   a. coins, why constant width?

   b. Reulaux triangle

Activity: make a square by folding a rectangular piece of construction paper. Draw and cut Relaux triangle of the width equal to the side of the square, rotate within the square.
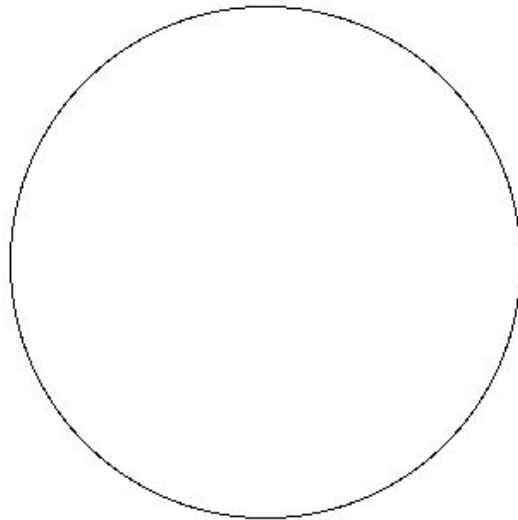
Activity: Solids of constant width (youtube)
http://www.youtube.com/watch?v=jYf3nOYM_mQ

Activity: rotary engine (youtube)
http://www.youtube.com/watch?v=yyrHeFpnaCg&NR=1

9. Problem: how to find a center and radius of a circle?

Activity: plot a cord, use compass to construct the
perpendicular through the center of the cord, this cuts the circle
into two half circles, this cord is the diagonal. Find its center
by constructing the perpendicular through the center.

10. Cycloid and the tautochrone and brachistochrone problems
http://www.youtube.com/watch?v=bUAbGXiBZng&NR=1&fe
ature=fvwp
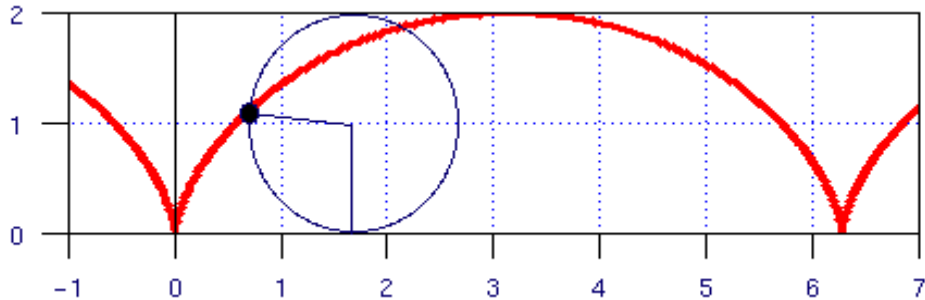
Activities: computer simulations, experiments.

More about a circle:

Star Trek Theorem
(a) Sum of angles in a triangle
(b) Full angle and half full angle
(c) Angles in equilateral triangle
(d) Angles in an isosceles triangle
(e) Star Trek theorem
(f) Angles based on the diameter

11. Homework: Plot a circle passing through three points, not
    all three on the same line: